$B \to X_s\gamma$ and $B \to K^*\gamma$ in the standard and 2H models

Marco Ciuchini†

INFN, Sezione Sanità,
V.le Regina Elena 299, 00161 Roma, Italy

Abstract

Theoretical predictions for the branching ratios of the $B \to X_s\gamma$ and $B \to K^*\gamma$ decays are calculated in the Standard Model and in the (type II) two-Higgs-doublet model. Both the complete leading and the partially known next-to-leading order QCD corrections are included. The uncertainties due to the regularization scheme dependence introduced by the incomplete NLO terms are discussed. The results are compared with the recent CLEO II measurements and a new lower limit on the charged Higgs boson mass, $M_{H^\pm} >\sim 200$ GeV, is obtained.

We calculate the theoretical predictions of the branching ratios for the decay $B \to X_s\gamma$ and $B \to K^*\gamma$ in the Standard Model and the type II two-Higgs-doublet model [1]. It is well known that the leading order (LO) QCD corrections are important [2], almost doubling the amplitudes of these decays. We include QCD correction using the results of ref. [3] to compute the relevant Wilson coefficient, $C_7^{eff}(\mu)$. We also include those next-to-leading (NLO) corrections which are already known [4, 5], as explained in ref. [6]. This results in a significant reduction of the dependence of $C_7^{eff}(\mu)$ on the renormalization scale μ, which is the main source of theoretical uncertainty in the leading order calculation [7], see figure 1. However this procedure is not consistent theoretically and, in fact, an unphysical regularization scheme dependence is introduced in the physical predictions in this way. We try to cope with this problem by considering two different cases, the $\overline{\mathrm{MS}}$ 't Hooft-Veltman (HV) and naive dimensional (NDR) regularization/renormalization schemes, taking, for each prediction, the mean value over the two schemes as the physical result. Moreover, the difference between them is assumed as a systematic error associated to our ignorance of the full next-to-leading corrections. This error is presented along with the usual one, due to the variation of the relevant parameters, Λ_{QCD} and m_t.

Figure 1. LO and NLO C_7^{eff} as a function of μ.

The relevant formulae to calculate the branching ratios we are interested in are

$$BR(B \to X_s\gamma) = \left[\frac{\Gamma(B\to X_s\gamma)}{\Gamma(B\to X l\nu_l)}\right] BR(B \to X l\nu_l),$$
$$\left[\frac{\Gamma(B\to X_s\gamma)}{\Gamma(B\to X l\nu_l)}\right] = \frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2}\frac{\alpha_e}{6\pi g(m_c/m_b)} F |C_7^{eff}(\mu)|^2,$$
$$g(z) = 1 - 8z^2 + 8z^6 - z^8 - 24z^4\ln(z),\ F = \frac{K(m_t/M_W,\mu)}{\Omega(m_c/m_b,\mu)}.$$

† E-mail: ciuchini@vaxsan.iss.infn.it.

Parameter	Value
$\lvert V_{ts}^{*}V_{tb}\rvert^2/\lvert V_{cb}\rvert^2$	0.95 ± 0.04
m_c/m_b	0.316 ± 0.013
m_t (GeV)	174 ± 17
λ_1 (GeV^2)	-0.15 ± 0.15
λ_2 (GeV^2)	0.12 ± 0.01
$m_b(\mu = m_b)$ (GeV)	4.65 ± 0.15
$F_1(0)$	0.35 ± 0.05
$BR(B \to Xl\nu_l)$	0.107 ± 0.005
$\Lambda_{QCD}^{n_f=4}$ (MeV)	330 ± 100
μ	$m_b/2$–$2m_b$

Table 1. Values of the parameters used to predict the radiative B decay rates.

$$BR(B \to K^*\gamma) = \left[\frac{\Gamma(B\to K^*\gamma)}{\Gamma(B\to X_s\gamma)}\right]\left[\frac{\Gamma(B\to X_s\gamma)}{\Gamma(B\to Xl\nu_l)}\right] BR(B \to Xl\nu_l)$$

$$\left[\frac{\Gamma(B\to K^*\gamma)}{\Gamma(B\to X_s\gamma)}\right] = \left(\frac{M_b}{m_b}\right)^3 \left(1-\frac{M_{K^*}^2}{M_B^2}\right)^3 \frac{|F_1(0)|^2}{1+(\lambda_1-9\lambda_2)/(2m_b^2)}$$

$BR(B \to X_s\gamma)$ includes also the known next-to-leading corrections to the matrix element, while non-perturbative $1/m_b^2$ corrections are included in $BR(B \to K^*\gamma)$. The numerical values of the different quantities appearing in these expressions are given in table 1. For more details on their choice, see ref. [6].

Using the previous formulae, we calculate the branching ratios in table 2. The errors shown in this table are due to the uncertainties on Λ_{QCD} and m_t. Combining the NLO results in HV and NDR for different values of μ, we obtain our final predictions in the Standard Model

$$\begin{aligned} BR(B \to K^*\gamma) &= (4.3 \pm 0.9^{+1.4}_{-1.0}) \times 10^{-5} \\ BR(B \to X_s\gamma) &= (1.9 \pm 0.2 \pm 0.5) \times 10^{-4} \\ \frac{\Gamma(B \to K^*\gamma)}{\Gamma(B \to X_s\gamma)} &= 0.23 \pm 0.09, \end{aligned}$$

Comparing them with the recent measurements [8, 9]

$$\begin{aligned} BR(B \to K^*\gamma) &= (4.5 \pm 1.5 \pm 0.9) \times 10^{-5} \\ BR(B \to X_s\gamma) &= (2.32 \pm 0.51 \pm 0.29 \pm 0.32) \times 10^{-4}, \end{aligned}$$

	$BR(B \to X_s\gamma) \times 10^4$		
μ (GeV)	LO	NLO$_{HV}$	NLO$_{NDR}$
$m_b/2$	3.81 ± 0.47	1.92 ± 0.19	2.77 ± 0.32
m_b	2.93 ± 0.33	1.71 ± 0.18	2.25 ± 0.25
$2m_b$	2.30 ± 0.26	1.56 ± 0.17	1.91 ± 0.21
	$BR(B \to K^*\gamma) \times 10^5$		
μ(GeV)	LO	NLO$_{HV}$	NLO$_{NDR}$
$m_b/2$	6.9 ± 1.5	4.4 ± 0.8	6.4 ± 1.3
m_b	5.3 ± 1.1	3.8 ± 0.8	5.0 ± 1.0
$2m_b$	4.2 ± 0.9	3.3 ± 0.7	4.1 ± 0.8

Table 2. Theoretical predictions of the radiative branching ratios.

Figure 2. Predictions for $BR(B \to X_s\gamma)$ in the 2H model with $\tan\beta = 2$ are given as a function of $M_{H^\pm}$. The experimental band is delimited by the dotted lines.

a very good agreement is found. Notice, however, that the estimate of the exclusive branching ratio strongly depends on the value assumed for the form factor $F_1(0)$.

Finally, let us consider the two-Higgs-doublet model known in the literature as Model II [1]. Two more free parameters are present in this model, $M_{H^\pm}$ and $\tan\beta$. The charged Higgs boson exchange only modifies the initial conditions of the Wilson coefficients. Moreover, for $\tan\beta > 1.5-2$, these become practically independent of $\tan\beta$. In figure 2, the $BR(B \to X_s\gamma)$ is reported as a function of $M_{H^\pm}$ for $\tan\beta = 2$. The band accounts for the theoretical uncertainties. The comparison with the experimental result gives a limit on $M_{H^\pm} >\sim 200$ GeV.

Acknowledgments

It is a pleasure to acknowledge the friendly collaboration of E. Franco, G. Martinelli, L. Reina and L. Silvestrini.

References

[1] S. Glashow and S. Weinberg, Phys. Rev. **D15** (1977) 1958; L.F.Abbott, P. Sikivie and M.B. Wise, Phys. Rev. **D21** (1980) 1393.
[2] B. Grinstein, R. Springer and M.B. Wise, Phys. Lett. **B202** (1988) 138.
[3] M. Ciuchini et al., Phys. Lett. **B316** (1993) 127; M.Ciuchini et al., Nucl. Phys. **B421** (1994) 41.
[4] A.J. Buras et al., Nucl. Phys. **B400** (1993) 37; *ibid.* **B400** (1993) 75.
[5] M. Ciuchini et al., Nucl. Phys. **B415** (1994) 403.
[6] M. Ciuchini et al., Phys. Lett. **B334** (1994) 137.
[7] A.J. Buras et al., MPI-Ph/93-77, TUM-T31-50/93.
[8] R. Ammar et al. (CLEO Collaboration), Phys. Rev. Lett. **71** (1993) 674.
[9] E.H. Thorndike (CLEO Collaboration), these proceedings.